

# Thermopower in the Quantum Hall Regime

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We consider the effect of disorder on the thermopower in quantum Hall systems. For a sample in the Corbino geometry, where dissipative currents are not carried by edge states, we find that thermopower behaves at high temperatures like a system with a gap and has a maximum which increases as the temperature is reduced. At lower temperatures this maximum reduces as a function of temperature as a result of tunneling across saddle points in the background potential. Our model assumes that the mean saddle point height varies linearly with the deviation in filling factor from the quantized value. We test this hypothesis against observations for the dissipative electrical conductance as a function of temperature and field and find good agreement with experiment around the minimum.

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Theoretical analysis of heat transport and thermopower in quantum Hall systems has concentrated on homogeneous systems in which the response is that of free quasiparticles or quasiholes and in which impurities are treated (if at all) as point-like scatterers [1–3]. In such homogeneous electron fluids, the thermopower is related to the entropy per charge [1, 3]. This information may be hard to extract experimentally given that the charge and heat currents are known to be greatly affected by edge contributions which may be different for heat and charge transport. However, measurements in the Corbino geometry are expected to get round many of the complications associated with inhomogeneous current distributions [2].

Localized puddles of compressible fluid within the quantum Hall state were predicted theoretically [4] and observed directly in scanning tunneling measurements [5]. While the quantized Hall response is that of the percolating fluid, the dissipative response is thought to be dominated by the motion between compressible puddles. Excitations in local equilibrium in one compressible puddle are transferred through the incompressible region to a neighboring puddle across saddle points in the potential. The average saddle point height is  $\Delta_s/2$ , where  $\Delta_s$  is the energy to nucleate a particle and hole near the saddle point. This energy controls the thermally activated dissipative conductivity at the center of a quantum Hall plateau which varies as  $e^{-\Delta_s/2T}$  in the absence of quantum tunneling [6]. When tunneling through saddles is important, the dissipative conductance is larger, with an apparent activation energy smaller than  $\Delta_s/2$  [7].

Here we show that the thermopower is controlled by the response at saddle points (see Fig. 1). Even in the presence of a percolating incompressible region, thermal excitations will be predominantly located in the compressible regions where there is no gap. The dissipative response of quantum Hall systems is then controlled by the response of such excitations to temperature and voltage differences across saddles. The link with the entropy per charge of the system, expected for homogeneous sys-

tems, is not simple in this context. The connection of the thermopower to entropy follows from the basic thermodynamics of the incompressible fluid, while the dissipative response in the inhomogeneous systems is determined by the coupling at saddle points in the background potential of separate reservoirs of localized excitations assumed to be in local equilibrium.

The case of transport occurring as a result of a chemical potential difference (voltage drop) was treated in [6] without taking account of tunneling, which was then discussed in [7]. A temperature difference across a saddle point would also lead to a flow of charge. The rate of transfer of quasiparticles with charge,  $-qe$ , across a saddle, shown moving left to right in Fig 1, is

$$i_{lr} = \frac{1}{h} \int_0^{\Delta_s} dE \mathcal{T}(E - E_{sp}) e^{-(E - \mu_l)/T_l}. \quad (1)$$

Here  $T_l$  and  $\mu_l$  are the temperature and chemical potential in the left puddle,  $\mathcal{T}$  is the transmission probability for a particle across the saddle and  $E_{sp}$  is the height of the saddle for quasiparticles.  $E_{sp}$  can vary between 0 and  $\Delta_s$ , where  $\Delta_s$  is the saddle point gap. Excitations with energy  $\Delta_s$  (or higher) would correspond to qp's having crossed the incompressible fluid and localizing as a 'minority' carrier above the neighboring qh-rich region (see Fig 1). Such processes would imply a connection between qp- and qh- channels and would need to be taken into account at high temperatures.

When the incompressible region is wide and there is negligible tunneling across the saddle, excitations only cross when  $E > E_{sp}$  so that  $\mathcal{T} = \theta(E - E_{sp})$ , where  $\theta$  is the Heaviside function. In this limit, the integral in (1) can be evaluated analytically. If  $\Delta_s \gg T_l$ , the result is  $i_{lr} = \frac{T_l}{h} \exp(-E_{sp}/T_l + \mu_l/T_l)$ . Taking  $\mu_l = -\mu_r = (-qe)\delta V/2$  and  $T = (T_l + T_r)/2$ , the net (number) current of quasiparticles crossing the saddle is

$$i_s = \frac{1}{h} \left( -(qe)\delta V + \left( 1 + \frac{E_{sp}}{T} \right) \delta T \right) e^{-E_{sp}/T}. \quad (2)$$

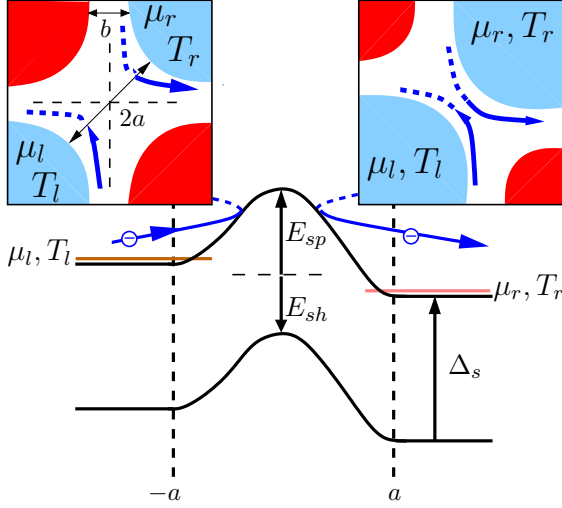


Figure 1. Band alignment and particle flow across a typical saddle point. Upper left: At the minimum of the dissipative conductance ( $\nu = \nu_m$ ,  $\delta\nu = 0$ ) the typical saddle point heights,  $E_{sp} = E_{sh} = \Delta/2$ , and the saddle point widths are equal. For  $\delta\nu > 0$  (upper right) the typical saddle points for (blue) qp-rich regions will grow and the saddle point width narrow while the (red) qh-rich regions shrink. Transfer across the saddle is via thermally activated tunneling. The net transfer reflects the potential and the temperature differences,  $(\mu_r - \mu_l)$  and  $(T_r - T_l)$ , across the saddle. In a thermopower measurement there is no net flow.

We will assume that the saddle point heights are randomly and symmetrically distributed about the mean, and that the net electrical and thermal conductivity of the network from quasiparticle transport is given by taking the average saddle point height,  $E_{sp}^{av}$ . As shown in [8], this is always the case for any network in which the log of the conductivity (thermal or electrical) is evenly distributed about the mean. We will come back to this point at the end. Adding the contribution of the corresponding network for quasiholes, which have charge  $+qe$ , the electric current is related to  $\delta V$  and  $\delta T$  via

$$I = L^{(11)}\delta V - L^{(12)}\frac{\delta T}{T}. \quad (3)$$

In the Corbino geometry the quantities,  $L^{(11)}$  and  $L^{(12)}$ , are scalars, as the edge currents and bulk Hall current make no net contribution to radial transport, and are given by

$$\begin{aligned} L^{(11)} &= \frac{(qe)^2}{h} (e^{-E_{sp}^{av}/T} + e^{-E_{sh}^{av}/T}) \\ \frac{L^{(12)}}{T} &= \frac{(qe)}{h} \left[ \left(1 + \frac{E_{sp}^{av}}{T}\right) e^{-E_{sp}^{av}/T} - \left(1 + \frac{E_{sh}^{av}}{T}\right) e^{-E_{sh}^{av}/T} \right]. \end{aligned} \quad (4)$$

The thermopower,  $Q$ , is minus the ratio of the voltage drop to the temperature difference when there is no net

transfer of charge from one edge to the other. In units of  $k_B/qe$  this gives  $Q = -\frac{1}{T} \frac{L^{(12)}}{L^{(11)}}$ , and

$$Q = -\frac{\left(1 + \frac{E_{sp}^{av}}{T}\right) e^{-E_{sp}^{av}/T} - \left(1 + \frac{E_{sh}^{av}}{T}\right) e^{-E_{sh}^{av}/T}}{e^{-E_{sh}^{av}/T} + e^{-E_{sp}^{av}/T}}. \quad (5)$$

At the center of a quantum Hall plateau, we expect  $E_{sp}^{av} = E_{sh}^{av} = \Delta_s/2$ , in which case  $L^{(12)}$  and the thermopower vanish. In this instance there are equal quasiparticle and quasihole flows from the hotter edge of a sample to the cooler edge and consequently no net transfer of charge.

If the filling fraction is increased (or decreased) from its value at the plateau center, the average height of the saddle points for quasiparticles will decrease (increase) while that for quasiholes will increase. In either case,  $L^{(12)}$  will be non-zero. We use the experimentally measured width of the plateau as a measure of the range in filling fraction/magnetic field over which the saddle point structure remains. We denote the filling fractions, at which the Hall conductance moves off its quantized value by  $\nu_m \pm \delta\nu_m$  and assume that, at these values, the corresponding average saddle point gap,  $E_{sh}^{av}$  (for  $\nu = \nu_m - \delta\nu_m$ ) or  $E_{sp}^{av}$  (for  $\nu = \nu_m + \delta\nu_m$ ), vanishes. Assuming that the variation is linear in the deviation  $g = \delta\nu/\delta\nu_m$  (we discuss this assumption at the end), gives

$$E_{sh}^{av} = (1 - g) \frac{\Delta_s}{2} \quad \text{and} \quad E_{sp}^{av} = (1 + g) \frac{\Delta_s}{2}. \quad (6)$$

Combining (5) and (6) we obtain a prediction for how the thermopower depends on temperature and magnetic field in the limit that  $\mathcal{T} = \theta(E - E_{sp}^{av})$ . Results are shown by the dashed lines in Fig. 2. As the temperature decreases the thermopower tends to  $Q \approx -1 - (1 - g)\Delta_s/2T$  for  $g > 0$  and  $Q \approx 1 + (1 + g)\Delta_s/2T$  for  $g < 0$ . Close to  $\delta\nu = 0$ ,  $Q$  changes by  $\Delta_s/T$  over a range in filling fractions  $\delta\nu \sim \delta\nu_m T/\Delta_s$  as  $T \rightarrow 0$ . A similar result has been obtained for a homogeneous integer quantum Hall system treated in the self-consistent Born approximation. The role of  $\Delta_s$  is played by the cyclotron energy reduced by a Landau level broadening parameter [2].

In most samples, particularly at filling fractions with weak quantum Hall states, the saddle point regions are not significantly wider than  $l_q$  and tunneling is important. For the special case of non-interacting particles, with potential energy near the saddle  $W = E_s - U_x x^2 + U_y y^2$ , the transmission probability is given by [9]

$$\mathcal{T}(E - E_s) = 1/(1 + e^{-\pi(E - E_s)/(\ell_q^2 \sqrt{U_x U_y})}). \quad (7)$$

This is provided that  $U_{x,y}/m\omega_c^2 \approx (\ell_q/a_{x,y})^2/2 < 1$ . The exponent in the denominator was found to be given correctly by the WKB approximation. We assume that (7) holds in the interacting case, with  $\Delta_s$  replacing the cyclotron energy  $\omega_c$ , and assume that, for a given sample,

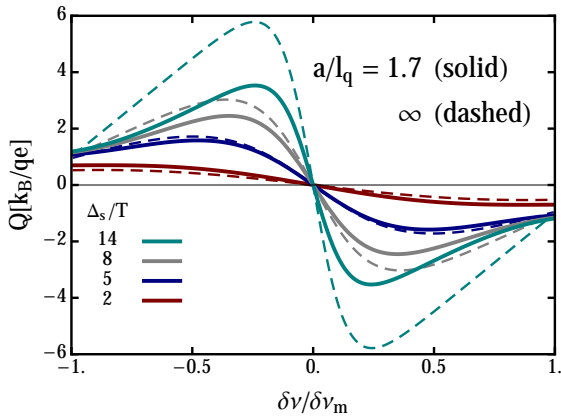


Figure 2. Thermopower for the Corbino geometry in the limit  $a/l_q \rightarrow \infty$  (dashed line) and for  $a/l_q = 1.7$  (solid) as a function of  $\delta\nu/\delta\nu_m$  for different values of  $\Delta_s/T$ . When  $a/l_q \rightarrow \infty$ , there is a discontinuity in  $Q$  at  $g = 0$ . Tunneling removes this and reduces the size of the maximum/minimum. The position of the maximum in  $|Q|$  and its value are shown in Fig. 3.

the parameter  $\sqrt{U_x U_y} = \Delta_s/a^2$  is the same for all saddles.

In Fig. 2 the solid lines show how the thermopower depends on  $\delta\nu/\delta\nu_m$  for the case  $a/l_q = 1.7$  for four different values of  $\Delta_s/T$ . Decreasing the temperature leads, initially, to an increased maximum absolute value for  $Q$  either side of  $\delta\nu = 0$ . The transport across a saddle point is still thermally excited. However, with tunneling there is significant transfer across the saddle point of qp/qh's with energies below the saddle point and the results mimic those for a system without tunneling but with a smaller gap. As the temperature is decreased, the proportion of the current from qp/qh states with energies below the saddle point increases. Below some temperature, which for  $a/l_q = 1.7$  is at  $T \sim \Delta_s/15$ , the thermopower starts to reduce. This is shown in Fig. 3(b), where the maximum of the absolute value of the thermopower,  $Q_{max}$ , as a function of  $\Delta_s/T$  is plotted for different values of  $a/l_q$ . In the Fig. 3(a), we show the value of  $\delta_{mx}$ , which is the value of  $\delta\nu/\delta\nu_m$  at which the thermopower has its maximum absolute value.

Fig. 3 shows a change in behavior as a function of temperature. At high temperatures the value and position of  $Q_{max}$  is that expected for a thermally activated system with  $Q_{max}$  increasing with decreasing  $T$ . Below a certain temperature,  $Q_{max}$  moves away from the plateau center, as shown in Fig. 3(a), and falls with decreasing temperatures. At very low temperatures ( $\Delta_s/T \gtrsim 50$  for  $a/l_q = 1.7$ ),  $Q_{max}$  tends towards the value set by the entropy per charge expected for a homogeneous system.

Our model assumes that stable fractionally charged ‘edge excitations’, localized on the puddles, are the only objects contributing to transport. However, at some fill-

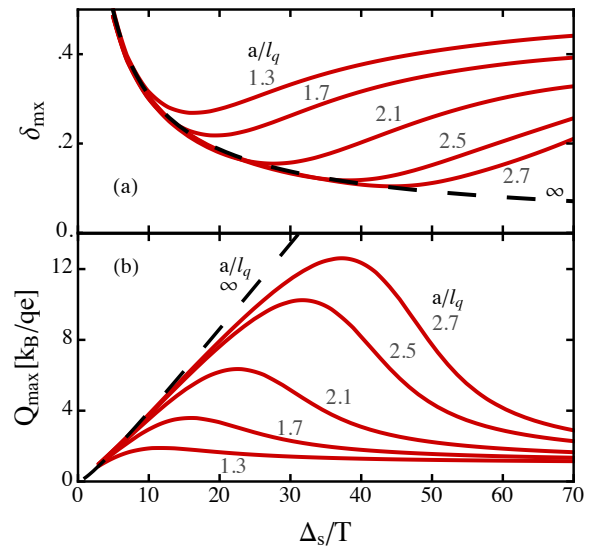


Figure 3. (a) The value of  $|\delta\nu/\delta\nu_m|$  at which the maximum in  $|Q|$  is observed,  $\delta_{mx}$ , and (b) the maximum of  $|Q|$  as a function of  $\Delta_s/T$ . Results are shown for different values of the parameter  $a/l_q$  (solid lines) and for  $a/l_q \rightarrow \infty$  (dashed lines).

ing fractions, neutral excitations are predicted to exist which would also be localized on each puddle. A microscopic study of excitations at  $\nu = 5/2$  suggested that the velocity for neutral excitations,  $v_n$ , is an order of magnitude less than that for charge excitations,  $v_c$ , for the case of a slowly varying background potential for the underlying electrons [10]. The barriers at saddle points in the potential would then be substantially less for neutral than for charged excitations and their effect on transport coefficients would become apparent only at much lower temperatures. If the neutral excitations are not strongly coupled to charge excitations, there should be no contribution to conductivity (obviously) or thermopower as the neutral excitations do not carry charge. They would provide an alternative channel for the transport of heat. Measuring the electronic contribution to all transport coefficients, and comparing thermopower, thermal conductivity and electrical conductivity might allow the different contributions to be separately identified.

We return now to the applicability of the model. It is valid for the temperature regime, in which phase coherence is not established between puddles and the excitations relax to local equilibrium in each puddle before reaching the next saddle point. We assume that the response of the network can then be computed from that of the average saddle point which, if the log of the saddle point responses is distributed evenly about a mean, is guaranteed by Dykhne’s theorem [8]. In the original treatment of conductivity, which does not take account of tunneling [6], the conductance at a saddle point is proportional to  $e^{-E_{sp}/kT}$ . As the heights of the saddle

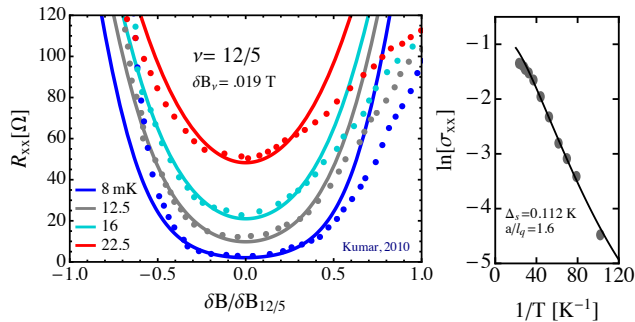


Figure 4. Right: Measured data points at  $\nu = 12/5$  from [12] and the theoretical fit for  $\log \sigma_{xx}$ . The experimental values for the resistance  $R_{xx}$  are transformed into a conductance measured in units of  $(qe)^2/h$  with  $q = 1/5$ . The theoretical parameters are the saddle point gap,  $\Delta_s$ , the width parameter,  $a$ , and the aspect ratio of the equivalent resistor network (see text). These parameters, together with an estimate of the width of the plateau as a function of magnetic field are used to predict the magnetic field dependence of the longitudinal resistance at different temperatures. The results are shown in the left panel. The data points are digitized from the curves in [12].

points,  $E_{sp}$ , reflect the variation of the background impurity potential and should be evenly distributed, the Dykhne theorem should apply.

Quantum tunneling modifies the response of the saddle so that its log is no longer exactly symmetrically distributed about the mean for the case of an even distribution of  $E_{sp}$ . This is because, when  $E_{sp,sh} \sim 0$  and the saddle point is close to disappearing, two adjacent qp/qh puddles are effectively shorted together by tunneling. In such cases the response is essentially independent of the height of the saddle point and temperature and it is better to think of an altered connectivity for the two (qp and qh) networks with the formation of a larger puddle and the removal of the saddle point. Empirically we find that the proportion of saddles, which act as shorts and open circuits, is at most 20% in the case of strong tunneling ( $l_q/a \lesssim 1$ ) [11]. Assuming that these shorts are distributed randomly throughout the network, their contribution to the resistance per square is negligible (the same number of ‘vertical’ and ‘horizontal’ links will be affected). For the case of thermal transport, even in the absence of tunneling, the distribution of log-conductances is not quite symmetric, but the effect is significant only in the cases where the saddle points are disappearing and should be accounted for via a change in connectivity.

We have also assumed that the average saddle point height depends linearly on the deviation of the magnetic field from its plateau center value. We can verify this by studying the dissipative conductance. We convert  $R_{xx}$  data into a conductance measured in units of  $2(qe)^2/h$  [6, 7] and from its temperature dependence we estimate the saddle point gap,  $\Delta_s$ , and the width parameter,  $a$ .

The absolute value of the conductance then gives an estimate of the aspect ratio of the resistance network or sample. The aspect ratio provides a check on the validity of the model. It should be the same for all filling fractions for a given sample and it should not be significantly different from one. Using the width of the plateau estimated from the Hall resistance data,  $R_{xy}$ , we predict the field dependence of the longitudinal resistance,  $R_{xx}$ , and compare with experiment. For the quantum Hall state reported in [12] at  $\nu = 12/5$ , we show values of  $\Delta_s$  and  $a$  and the measured and predicted dissipative conductance as a function of temperature in Fig 4. We assumed an aspect ratio (length to width ratio) of 1.4 for this sample. With these parameters and taking the width of the plateau,  $\delta B_{12/5}$ , from the traces of  $R_{xy}$  we predict the magnetic field dependence of  $R_{xx}$ . This is shown in the left hand panel and compared with experimental data points digitized from plots in [12]. The results show that for  $|\delta B/\delta B_{12/5}| \lesssim .4$  the agreement is good. The asymmetry between positive and negative  $\delta B$  is not included in our model and presumably relates to the nature of the excitations in the puddles which are affected by nearby competing quantum Hall states. The prefactor of  $\sigma_{xx}$  scales with the square of the excitation charge. At magnetic fields close to  $\nu = 12/5$ , excitations are likely to carry larger (smaller) charge at lower (higher) magnetic fields, which is consistent with what is observed.

We have shown that the thermopower of a quantum Hall system, which is accessible in the Corbino geometry, should be a non-monotonic function of both temperature and magnetic field close to the plateau center. It switches from thermally activated transport with an increasing maximum approaching the plateau center as the temperature decreases to a system dominated by tunneling. As the temperature decreases in this regime, the thermopower reduces slowly to the value expected of a homogeneous system set by the entropy per charge. This effect is a consequence of the slowly varying background potential induced by ionized donors set back from the 2DEG, and is intrinsic to any heterostructure. We have validated our model by studying the electrical response as a function of magnetic field. We find results for the dissipative conductance which agree quantitatively with experiment for  $|\delta B/\delta B_{12/5}| \lesssim 0.4$ .

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- [1] N. R. Cooper, B. I. Halperin, and I. M. Ruzin, Phys. Rev. B, **55**, 2344 (1997).
  - [2] Y. Barlas and K. Yang, Phys. Rev. B, **85**, 195107 (2012).
  - [3] K. Yang and B. I. Halperin, Phys. Rev. B, **79**, 115317 (2009).
  - [4] A. L. Efros, Solid State Comm., **67**, 1019 (1988).
  - [5] J. Martin, S. Ilani, B. Verdenne, J. Smet, V. Umansky,

- D. Mahalu, D. Schuh, G. Abstreiter, and A. Yacoby, *Science*, **305**, 5686 (2005).
- [6] D. G. Polyakov and B. I. Shklovskii, *Phys. Rev. Lett.*, **73**, 1150 (1994).
- [7] N. d’Ambrumenil, B. I. Halperin, and R. H. Morf, *Phys. Rev. Lett.*, **106**, 126804 (2011).
- [8] A. M. Dykhne, *Sov. Phys. JETP*, **32**, 63 (1971).
- [9] H. A. Fertig and B. I. Halperin, *Phys. Rev. B*, **36**, 7969 (1987).
- [10] X. Wan, Z.-X. Hu, E. H. Rezayi, and K. Yang, *Phys. Rev. B*, **77**, 165316 (2008).
- [11] N. d’Ambrumenil, (2012), unpublished.
- [12] A. Kumar, G. A. Csáthy, M. J. Manfra, L. N. Pfeiffer, and K. W. West, *Phys. Rev. Lett.*, **105**, 246808 (2010).